Spring 2004

Physics 137B Section 1: Problem Set #4
Due: 5PM Friday Feb 19 in the appropriate dropbox inside 251 LeConte (the "reading room")

Suggested Reading for this Week:

- Bransden and Joachain (B& J) section 8.2
- Hyperfine splitting is not discussed in B& J. The treatment in class is identical to Griffiths section 6.5

Homework Problems:

1. Use first order perturbation theory to find the relativistic correction to the ground state energy of a one dimensional simple harmonic oscillator.

2. (Liboff Problem 13.15) Consider the quantum mechanical dumbbell from problem 5 of Problem set 3. If both ends are equally charged, the rotating dumbbell has angular momentum $\vec{L}$ and the corresponding magnetic dipole moment is
$$\vec{\mu} = \frac{e}{2mc}\vec{L}$$
where e is the net charge of the dipole. The interaction energy between this magnetic dipole and a constant uniform magnetic file $\vec{B}$ is
$$H' = -\vec{\mu} \cdot \vec{B}$$
   (a) If $\vec{B}$ points in the z direction, show that H' separates the $(2\ell+1)$-fold degenerate E_ℓ energies of the rotating dipole
   (b) Apply these results to find the splitting of the P states of the hydrogen atom (neglecting for now the spin-orbit couple).

   We will return to this phenomenon, *the Zeeman effect*, later this semester.

3. (Liboff Problem 12.6) In quantum mechanics, when one says that the vector $\vec{J}$ is conserved, one means that for any state the system is in, the expectation values of the 3 components of $\vec{J}$ are constant. This follows if these 3 operators all commute with H. Show that for the hydrogen atom with spin-orbit coupling $\vec{L}$ and $\vec{S}$ are not conserved but $\vec{J}$ is.

4. (Griffiths Problem 6.26) By appropriate modification of the hydrogen formula, determine the hyperfine splitting in the ground state of

   (a) Muonic hydrogen (in which a muon - same charge and g-factor as the electron but 207 times the mass - substitutes for the electron

   (b) positronium (in which a positron - same mass and g-factor as the electron but opposite charge - substitutes for the proton

   (c) Muonium (in which the antimuon - same mass and g-factor as a muon but opposite charge - substitutes for the proton and the muon substitutes for the electron)

   Hint: Don't forget to use the reduced mass when necessary. Note: The answer you get for positronium is quite far from the experimental value of 8.41×10^{-4} eV. This discrepancy is due to pair annihilation ($e^+e^- \to \gamma\gamma$)

5. (Griffiths Problem 6.33) Calculate the wavelength, in cm, of the photon emitted under a hyperfine transition in the ground state ($n = 1$) of deuterium. Deuterium is "heavy" hydrogen with an extra neutron in the nucleus. The proton and neutron bind together to form a deuteron with spin 1 and magnetic moment

$$\vec{\mu_d} = \frac{g_d e}{2m_d}\vec{S_e}$$

   The deuteron g-factor is 1.71.